7. (a) Show that : **2**

$$\tan\frac{\theta}{7}+\tan\frac{\theta+\pi}{7}+\ldots.\tan\frac{\theta+6\pi}{7}=7\tan\theta.$$

(b) If $\tan(\theta+i\phi)=\sin(x+iy)$, prove that $\coth y.\sinh 2\phi=\cot x.\sin 2\theta$. **2.5**

Section IV

8. (a) Solve the equation : **2**

$$\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}.$$

(b) Separate into real and imaginary parts :

$$\cosh^{-1}(x+iy).$$ **2.5**

9. (a) Sum the series $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+\ldots.$ to n terms and deduce the sum to infinity. **2**

(b) Sum the series :

$$\cos\theta-\frac{1}{2}\cos 2\theta+\frac{1}{3}\cos 3\theta-\ldots.\infty.$$ **2.5**

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No. of Printed Pages : 04 **Roll No. ....**

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B.A. & Hons. (Subsidiary)

EXAMINATION, 2025

(Second Semester)

(Re-appear Only)

MATHEMATICS

Paper BM-121

Number Theory and Trigonometry

Time : 3 *Hours*] [*Maximum Marks* : 27

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Show that $(2^{4n} - 1)$ is divisible by 15. **2.5**

(b) If a/bc and $(a, b) = 1$, then a/c. **2**

(c) If $x = \cos\theta + i\sin\theta$, then find $x^n - \frac{1}{x^n}$. **2**

(d) Find the general value of Log(–5). **2.5**

Section I

2. (a) If a, m, n are non-zero integers, then $(a, m, n) = 1$ if and only if $(a, m) = 1$ and $(a, n) = 1$. **2**

(b) State and prove Euclid's second theorem. **2.5**

3. (a) Solve the congruence $222x \equiv 12 \pmod{18}$. **2**

(b) State and prove Fermat's theorem. **2.5**

Section II

4. (a) Prove that $\phi(n) = \frac{n}{2}$ iff $n = 2^k$ for some integer $k \geq 1$. **2.5**

(b) If x is any real number, then $\left[\frac{[x]}{n}\right] = \left[\frac{x}{n}\right]$, where n is a positive integer. **2**

5. (a) Find all n such that $d(n) = 10$. Hence find the least such value of n. **2**

(b) If p is an odd prime, then $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}}$. **2.5**

Section III

6. (a) If α, β be the roots of $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$.

(b) If $z = x + iy$, where x and y are real, find the real and imaginary parts of $\frac{\cos z}{z+1}$. **2.5**